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The large probability of the 0^+ ground states

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Abstract. We investigate the dominance of 0^+ states as the lowest states in shell model calculations with random two-body interactions in a single j-shell. We have found an explanation of the large probability of the 0^+ ground state.

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The spins of even-even nuclei are always 0^+ without any exception. This fact is believed to be a consequence of the strong attractive short-range interaction. However, Johnson, Bertsch and Dean discovered an extremely interesting phenomenon [1]. This is the dominance of 0^+ states as the lowest states in shell model calculations with random two-body interactions. After this discovery, many works have been accumulated to understand this fact [2–10].

In this paper, we present our study of this problem taking a simple system such as four particles in a single j-shell. We assumed the Box-Muller method to produce random Gaussian two-body interactions. In these calculations j runs from 7/2 to 31/2. We confirmed the dominance of 0^+ states as ground states. More precisely, for j larger than 15/2, the probability for 0^+ states to be the ground states is confirmed to be always the largest one, as shown in fig. 1. In fig. 2 the probability of the spin I to be the ground state is shown for 5-particle systems with different j. The $I = j$ states are very likely to be the ground states.

In order to understand the reason why 0^+ and $I = j$ states have large probabilities as the ground states, we expand the expectation values of the Hamiltonian H in terms of the two-body interaction strengths G_J as follows:

$$E_{I,v,\beta} = \sum_J \alpha^J_{I,v,\beta} G_J ,$$

where I is the spin of a state, v its seniority and β its additional quantum number. The coefficients α for $j = 9/2$ with four particles are shown in table 1. We find a good correspondence between the probability of the state of spin I to be the ground state and the value of α. If α^J_I is the largest coefficient among $\alpha^J_{I'}$, the probability is large. The "pred" in table 1 means that the probability is calculated by using the following formula:

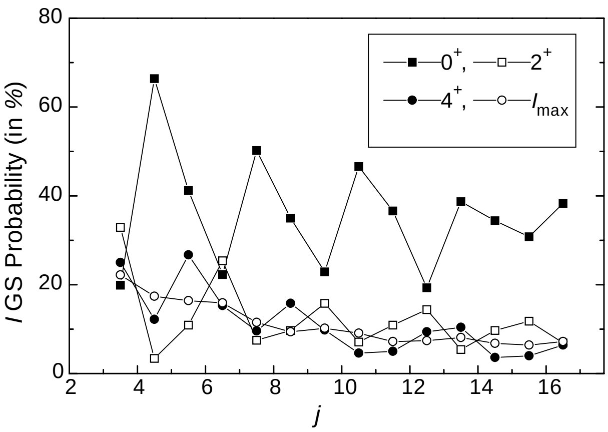

Fig. 1. The IGS probability (large probability of finding I to be the angular momentum of the ground state) for 4-particle systems with different j.

$$\int \mathrm{d}G_0 \int \mathrm{d}G_2 \cdots \int \mathrm{d}G_8 \int \mathrm{d}E_{0,0} \cdots \int_{E_{0,0}} \mathrm{d}E_{12,4}$$
$$\times \delta\left(E_{0,0} - \sum_J \alpha^J_{0,0} G_J\right) \cdots \delta\left(E_{12,4} - \sum_J \alpha^J_{12,4} G_J\right)$$
$$\times \rho(G_0)\,\rho(G_2)\,\rho(G_4)\,\rho(G_6)\,\rho(G_8)\,,$$

where $\rho(G_J) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}G_J^2\right]$ and $E_{I,v}$ is the energy of the state with spin I and seniority v.

Until now we did not take into account mixtures among states with the same I. The probabilities shown

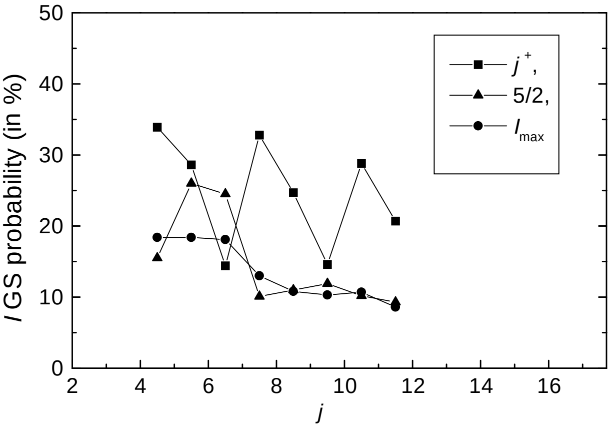

Fig. 2. The IGS probabilities for 5-particle systems with different j.

Table 1. The coefficients $\alpha^J_{I,v,\beta}$ for each angular momentum state in the case of $j = \frac{9}{2}$ and $n = 4$. Bold (italic) font is used for the largest (smallest) $\alpha^J_{I,v,\beta}$. The column "test" is obtained by the running the two-body random interactions. The last column, "pred"(prediction), is obtained by integrating over the distribution functions of the ensemble and using the approximation $E_{I,v,\beta} = \sum_J \alpha^J_{I,v,\beta} G_J$, refer to the text for details. When a state cannot be uniquely labeled by its angular momentum, only total probability of IGS is presented in the first one, and probabilities of other I-angular momentum states to be the ground states are labeled using star symbols in the column "test".

I	G_0	G_2	G_4	G_6	G_8	test (%)	pred (%)
0	**1.60**	0.50	0.90	1.30	1.70	66.4	14.150
0	0.00	0.20	**2.57**	**2.91**	*0.32*	*	30.649
2	0.60	1.43	1.22	0.893	1.86	3.7	1.844
2	0.00	1.35	1.69	1.70	1.26	*	1.260
3	0.00	0.36	2.28	2.63	0.71	0	0.110
4	0.00	**2.04**	1.02	*0.890*	2.06	11.8	18.852
4	0.00	0.50	2.08	2.43	0.99	*	0.000
4	0.60	0.68	1.04	2.40	1.28	*	3.540
5	0.00	1.00	1.59	1.84	1.57	0	0.00
6	0.60	0.34	1.66	1.33	2.07	0	2.103
6	0.00	1.64	0.98	1.08	2.29	0	0.000
6	0.00	0.39	1.85	2.34	1.43	0	0.000
7	0.00	1.20	1.09	1.40	2.31	0	0.000
8	0.60	0.55	0.68	1.58	2.59	0.2	0.030
8	0.00	0.41	1.42	2.05	2.13	*	0.000
9	0.00	0.17	1.33	2.12	2.38	0	0.000
10	0.00	0.70	0.69	1.41	3.21	0	0.176
12	0.00	*0.00*	*0.52*	1.69	**3.78**	17.9	27.275

in the column "test" are obtained by taking into account the mixing. We find that the probabilities shown as "test" become larger than those shown as "pred". This is quite reasonable, because a 0^+ state is pushed down by the mixing.

We use here a way to avoid the effect of the mixing in the space of states with the same spin I. The trace of H is independent of the mixing. We calculate the probability for an average over the energies of $E_{I,v,\beta}$ to be the lowest one. Here the average can be calculated by taking the trace. The result is shown in table 2 for $j = 31/2$ with four particles. Now the largest probability to be the ground state is found for the highest spin $I = 56^+$. The probability of the 0^+ average energy to be the lowest one comes next. However, we should be aware that there is only one state for $I = 56^+$. On the other hand, there are five 0^+ states. Some of them are pushed down far from their average. Thus we can expect that the probability of a 0^+ state to be the ground state is larger than that of the $I = 56^+$ state. It is interesting to note that some of the α's for 0^+ and $I = 56^+$, take the largest values for a specific interaction G_J.

Table 2. The IGS probability (up to the fifth largest cases) and IGS average probability for $j = 31/2$ with four particles. The IGS average probabilities are calculated in terms of the trace of the Hamiltonian H in each angular momentum I. The S_E is the standard deviation defined by $\langle (E_{I,v,\beta})^2 - \langle E_{I,v,\beta}\rangle^2\rangle^{1/2}$.

I	IGS probability (%)	IGS average probability (%)	S_E
0	30.8	11.5	3.06
2	11.8	3.7	2.69
4	4.0	0.4	2.47
6	7.6	1.1	2.37
56	6.4	23.3	0.00

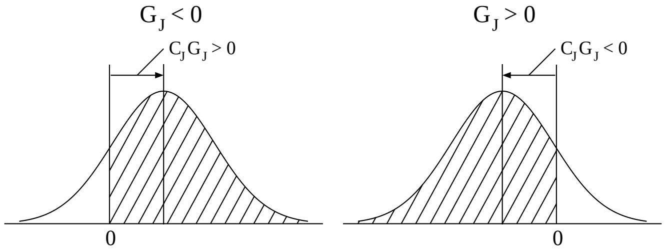

Fig. 3. The probability of ΔE.

We try to explain why a state has a large probability to be the ground state when some $\alpha^J_{I,v,\beta}$ of this state are the largest among $\alpha^J_{I',v',\beta'}$. Let us look at the difference between $E_{I',v',\beta'}$ and $E_{I,v,\beta}$:

$$\Delta E = E_{I',v',\beta'} - E_{I,v,\beta} = C_J G_J - F\,,$$

where $C_K = \alpha^K_{I',v',\beta'} - \alpha^K_{I,v,\beta}$ and $F = \sum_{J' \neq J} C_{J'} G_{J'}$. It should be noticed that C_J is negative definite, and $C_J G_J$ is either negative or positive definite depending on the sign of G_J. When $G_{J'}$ takes a random number produced by the Gaussian distribution, F, too, takes a Gaussian distribution. Figure 3 explains why ΔE has a large probability (shown by shadow) to be positive (namely for $E_{I,v,\beta}$ to be the lowest energy) when $G_J < 0$. This is an explanation of the question raised above. We have thus found an explanation of the large probability of the 0^+ ground state. The question why several α^J_0 take the largest values among the coefficients α_I^J ($I \neq 0$) is currently studied by the randomness of two-body coefficients of fractional parentage [11].

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